Miller Fellow Focus: Bojko Bakalov

Second year Miller Fellow Dr. Bojko Bakalov studies novel mathematical structures motivated by theoretical physics. Dr. Bakalov’s research is mainly in algebra but he has also done related work in geometry and topology. He is hosted by Professor Nicolai Reshetikhin in the Department of Mathematics.

Topology is a field of mathematics that studies shapes. In two dimensions, it is concerned with the possible shapes and deformations of surfaces. Let us consider only closed surfaces: spheres, doughnuts, or spheres with several handles attached, but also let us allow holes in them. For example, a sphere with two holes, i.e. two disks cut out from it, is the same as a cylinder. Such surfaces can be sewed together by gluing a hole from the first surface to a hole from the second one. It is easy to see that any surface can be obtained by sewing together disks and spheres with three holes (called “pairs of pants”). But this can be done in many different ways, and the question arises to describe all possible ways. This “lego game of surfaces” was studied by Dr. Bakalov and Professor Alexander Kirillov Jr. in their book “Lectures on Tensor Categories and Modular Functors” recently published by the American Mathematical Society. The “tensor categories” and “modular functors” are certain abstract algebraic objects arising naturally both in representation theory (which is a part of algebra) and in conformal field theory (a part of theoretical physics). In the book the authors also explore the relations of these structures to the topology of three-dimensional bodies. All this is a part of a complex and profound web that relates many subjects in mathematics and physics. As an historical remark, let us note that the start of this theory was motivated by work of the Miller Institute Executive Committee member Professor Vaughan Jones on the study of
knots (where Professor Reshetikhin has made important contributions as well).

The current research of Dr. Bakalov focuses on other algebraic structures, called “vertex algebras”, that were introduced in 1986 by recent Miller Professor Richard Borcherds. In contrast to usual algebra, where one has the operations of addition, subtraction and multiplication, vertex algebras have addition, subtraction, and infinitely many multiplications. All these infinite number of multiplications have to satisfy an infinite collection of complicated identities. To organize them in a neat form, one considers the generating function for the multiplications: a formal series having them as coefficients. In other words, instead of infinitely many multiplications now we have only one, which, however, depends on a parameter z. This parameter can be considered as the coordinate on a surface; thus, we can consider the multiplication as determined by a surface. More precisely, it is given by the “pair of pants”: the two legs correspond to the inputs, and the waist to the output. To other surfaces one can assign more general operations. Hence, the vertex algebras are also related to conformal field theory. Although widely known, this relationship is still partially conjectural, and Dr. Bakalov’s goal is to understand it deeper. Another aspect of Bakalov’s work concerns the purely algebraic study of vertex algebras and related structures called “conformal algebras”. The latter are the subject of a 140-page paper, joint with A. D’Andrea and V. G. Kac, which is to appear in Advances in Mathematics.

Bojko Bakalov can be reached at bakalov@math.berkeley.edu.


Awards and Honors

James Barber, Visiting Miller Professor (Spring ’01), of Imperial College in London was awarded the Flintoff Medal of The Royal Society of Chemistry.

Roger Green, Visiting Miller Professor (Fall ’94), of the University of Auckland in New Zealand, and Patrick Kirch, Miller Professor (’93–’94), of the University of California in Berkeley, collaborative project in historical reconstruction of an Ancient Polynesian culture dating 2000-2500 years ago was completed. Their project has been published in a book entitled “Hawaiki, Ancestral Polynesia. An Essay in Historical Anthropology”.

Sarah Otto, Miller Fellow (’92–’94), of the University of British Columbia in Canada was awarded the Steacie Fellowship from NSERC.

Jean Taylor, Visiting Miller Professor (Spring ’99), of Rutgers University, received an honorary doctorate from Mount Holyoke College.