The Adolph C. and Mary Sprague

Miller Institute

for Basic Research in Science

Newsletter
Winter 2025

Phase spaces, pendulums and periodic states

Miller Fellow Focus: Rohil Prasad

1. Introduction

I am a mathematician working in the areas of *dynamical systems* and *symplectic geometry*. Dynamical systems are mathematical models of physical systems that evolve over time. There are many examples that have captured our attention for hundreds, if not thousands, of years: motions of celestial bodies, flowing wind and rivers, and the cyclical fluctuations of predator/prey populations.

Symplectic geometry is relatively young and mysterious. It is a kind of geometry based on a notion of *two-dimensional area*. This is very different from the ruler-and- compass geometry taught in schools, where one can measure the lengths of lines, the angles between them, and the areas of triangles or other shapes. In symplectic geometry, the last measurement is the only permissible one. The unique properties of symplectic geometry make it difficult to understand. This is reflected in the historical record. Hermann Weyl named the field, inspired by the Greek *symplektikos* ("complex"), in 1939. Most tools that modern symplectic geometers use date back to only the 1980s.

I will begin by describing how mathematicians think about dynamical systems. Then, I will summarize some key historical developments that connected dynamical systems to symplectic geometry. After that, I will discuss my own

research. Berkeley features prominently in all of this. Several of the mathematicians that I will mention have been PhD students or faculty members here.

2. Phase spaces and pendulums

A (discrete) dynamical system is defined by a *phase space* and a *transformation*. Phase space is a record of all of the possible states of the system. For instance, consider a pendulum swinging on a string. Its phase space is a collection of pairs of numbers a, m . The number a is the *angle* of the pendulum and the number m records the *momentum* (**Figure 1**).

The transformation is a kind of function on phase space. For any input state, the output state is its evolution after some set amount of time (e.g. 1 second, 1 hour, 1 day). In the case of the pendulum and other mechanical systems, the

CONTINUED ON PAGE 2 >

INSIDE THIS EDITION

The Miller Institute is "dedicated to the encouragement of creative thought and the conduct of research and investigation in the field of pure science and investigation in the field of applied science in so far as such research and investigation are deemed by the Advisory Board to offer a promising approach to fundamental problems."

"What I appreciated most at the Miller Institute is the weekly interactions with fantastic scholars from other disciplines, including mathematics, physics, biology, astronomy, and computer science... During the 20-minute talk on Tuesday lunch, it's a great opportunity to escape from routine lab activities, more like taking a small scientific break during a long research marathon... Learning about different fields at the Miller Institute makes me recognize the beauty of other scientific fields and broadens my horizon way beyond the tunnel view of chemistry."

- Yao Yang
Assistant Professor
Department of Chemistry & Chemical Biology
Cornell University
Miller Fellow 2021-2024

> CONTINUED FROM PAGE 1 [Miller Fellow Focus]

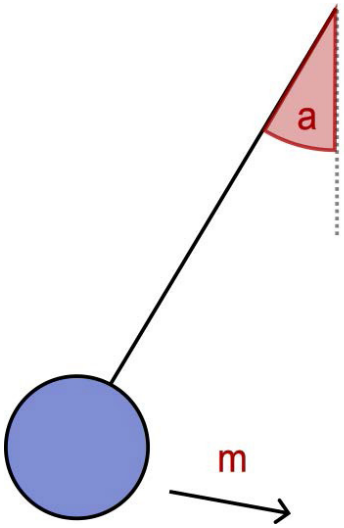

Figure 1. Angle and momentum of a pendulum.

transformation is derived from well-known physical equations. Given a pendulum in a state (a_0, m_0), there is an explicit formula that will compute its state (a_1, m_1) one second later.

3. Poincaré, Arnol'd, and Floer

The modern theory of dynamical systems was initiated by Henri Poincaré in the late 19th century, in a series of works on the three-body problem in celestial mechanics. Poincaré's many contributions from this time are impossible to summarize here, so I will focus on one that is particularly relevant to my work.

The *Poincaré–Birkhoff theorem* (the joint name acknowledges the equally important contributions of George Birkhoff) implies the three-body problem has many *periodic states*. These are states whose long-term evolution is as simple as possible: they return to themselves after a finite amount of time evolution. Moreover, the result is *robust*. Even for perturbations of the three-body system, there are still many periodic states.

In the 1960s, Vladimir Arnol'd made a visionary proposal that linked the Poincaré–Birkhoff phenomenon with symplectic geometry. For many dynamical systems, including the three-body problem, the phase space has a natural *symplectic structure*, and the transformation preserves this structure. We will refer to these as *symplectic dynamical systems*. Arnol'd proposed that for many symplectic systems, the number of periodic states is governed by the symplectic geometry of the phase space, rather than by the transformation. This explains the Poincaré–Birkhoff robustness: periodic states persist under perturbation because the phase space stays the same. The first cases of Arnol'd's proposal were verified in a breakthrough 1983 paper by Charles Conley and Eduard Zehnder. A subsequent breakthrough by Andreas Floer, inspired by Conley and Zehnder's work, confirmed many more cases only a few years later. Floer's method used a new tool now known as *Floer homology*. Floer homology and its variants continue to be the best tools available for finding periodic states of symplectic systems.

4. Periodic states of two-dimensional systems

Part of my PhD thesis studied periodic states of *two-dimensional* symplectic systems. These are symplectic systems whose phase space is two-dimensional. In joint work with Dan Cristofaro-Gardiner and Boyu Zhang, I showed that for a "typical" system of this kind, the periodic states are essentially everywhere in phase space. For any state, there is another arbitrarily close state that is periodic. Our result verified, in the 2D symplectic case, the tenth problem in Stephen Smale's "mathematical problems for the next century". In fact, there are two proofs. Oliver Edtmair and Michael Hutchings proved a restricted version of the result simultaneously and independently; a later note by Dan, Boyu, Daniel Pomerleano, and myself implied that their method works in full generality.

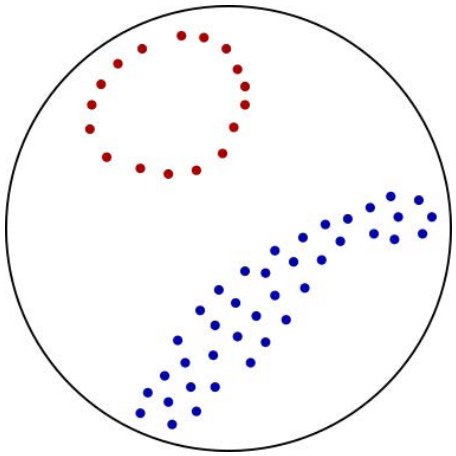

Figure 2. A schematic of the time evolutions of a periodic state and a non-periodic intransitive state.

The key to this result was symplectic geometry. Our proof built on a vast body of knowledge on two-dimensional symplectic spaces, spanning thousands of pages. Ideas of Masayuki Asaoka, Michael Hutchings (my Miller host), Kei Irie, Peter Kronheimer, Yi-Jen Lee, Tomasz Mrowka, and Clifford Taubes (my undergraduate mentor) were very important for us. Later, I proved a refined result: for a typical 2D symplectic system, the periodic states can be taken to evenly distribute throughout phase space.

5. Intransitive states

In my first year as a Miller fellow, I introduced some methods for detecting non- periodic states that still evolve over time in interesting ways. An *intransitive state*[1] is a state that always stays in

[1] A remark for mathematician readers: this definition is meant to oppose the more well-known *transitive states*, which have dense orbit in phase space.

> CONTINUED FROM PAGE 2 [Miller Fellow Focus]

some fixed region of phase space as it evolves over time. Periodic states, for example, are intransitive, since they evolve through a finite collection of points. However, many symplectic systems have intransitive states that are not periodic. There are simple physical examples, such as a pair of oscillating springs, whose periods of oscillation have an irrational ratio. See **Figure 2** for a schematic of periodic and intransitive states.

In a pair of recent preprints, the first joint with Dan Cristofaro-Gardiner, I showed that intransitive states are ubiquitous in low-dimensional symplectic dynamical systems, including 2D symplectic systems mentioned above. Our result in this case can be summarized as follows. Previously, we showed that a "typical" such system has periodic states throughout phase space. When we remove the "typical" assumption, this might not hold anymore. However, the intransitive states are still spread out throughout phase space! A particularly exciting aspect of these works is their generality: most results on intransitive states require significant additional assumptions.

The proofs of these results also rely on low-dimensional symplectic geometry. In addition to the names from the previous section, ideas of Joel Fish, Viktor Ginzburg, Basak Gürel, and Helmut Hofer (my PhD advisor) were important inspirations.

5. What's next

This research has opened up some new paths. One (ambitious) goal is to develop a theory for detecting "quasiperiodic" states of 2D symplectic systems, which could have several applications. Another goal is to extend my results to symplectic systems with higher-dimensional phase spaces, which would greatly widen the scope of the theory. The generous support of the Miller Institute and the Berkeley Mathematics Department have been invaluable for the work presented here and for this ongoing work.

Rohil Prasad is a second-year Miller Fellow in Mathematics, hosted by Prof. Michael Hutchings. He received his PhD in Spring 2023 from Princeton University under the guidance of Prof. Helmut Hofer. **He was recently awarded the 2024 Michael Brin Dynamical Systems Prize for Young Mathematicians.**

Contact: rrprasad@berkeley.edu

Miller members celebrated the end of summer together with a picnic in Codornices Park in Berkeley

In the News

(see more current & past Miller Institute news: **miller.berkeley.edu/news**)

The 2025 Steele Prize for Seminal Contribution to Research was awarded to **Ken Ribet** (Miller Professor Fall 1990, Miller Senior Fellow 2022-present) for his 1976 *Inventiones Mathematicae* paper, "A modular construction of unramified p-extensions of $Q(\mu_p)$."

David Britt (Visiting Miller Professor 2023) was recognized by the NC State University College of Sciences with the **Distinguished Alumnus Award** for his "exceptional achievements and service [that] have brought honor and distinction to the college."

Aditi Lahiri (Visiting Miller Professor 2014) was awarded a prestigious **Synergy Grant by the European Research Council (ERC)**. She has also won three Advanced Investigator Grants.

Gabriel Orebi Gann (Miller Professor 2021-2022) won an **Early Career award from the ICFA** (International Committee for Future Accelerators).

Jonathan Wolf (Miller Fellow 2024-2027) was a **2024 recipient of the Study of the Earth's Deep Interior Section Award for Graduate Research** from the American Geophysical Union (AGU).

Roland Bürgmann (Miller Professor 2014, Executive Committee 2016-2020) was a **2024 recipient of the William Bowie Lecture Award** from the American Geophysical Union (AGU).

Gibor Basri (Miller Professor 1997-1998) was honored with the **2024 Arthur B.C. Walker II Award from the Astronomical Society of the Pacific** for outstanding achievements in astronomy and education by an African-American scientist.

Iwnetim Abate (Miller Fellow 2021-2024) won **ISSI Young Scientist Award for energy materials** for his "pioneering work on advanced materials in renewable energy technologies." His research group was awarded the Department of Energy, **ARPA-E Award** for extracting hydrogen from rocks. Also, **Abate** was awarded the **Professor Amar G. Bose Research Grant** to "support bold research projects across diverse areas of study, including a way to generate clean hydrogen from deep in the Earth..."

NPR featured Michael Manga (Miller Professor 2008-2009, 2024-2025) and his group's research papers about geysers and earthquakes on their homepage in an article entitled "When Steamboat goes WHOOSH, scientists look for answers."

Emeritus Miller Institute Members!

Do you wish to receive the Miller Institute newsletter at your home address? Please email millerinstitute@berkeley.edu with your updated address information so as not to miss a single issue.

Miller Research Competitions: Awards

On December 3, 2024, the Advisory Board of the Miller Institute met to select next year's Professorship awards. The Board is comprised of four advisors external to UCB: Scott Edwards (Evolutionary Biology, Harvard), Anna Gilbert (Math, Statistics & Data Science, Yale), Linda Hsieh-Wilson (Chemistry, Caltech) and Eliot Quataert (Astrophysical Sciences & Astronomy, Princeton); and four internal Executive Committee members: Executive Director Chung-Pei Ma (Astronomy & Physics), Jeffrey Long (Chemistry & Chemical & Biomolecular Engineering), Nicole King (Molecular & Cell Biology) and Alistair Sinclair (Computer Science). The Board is chaired by Chancellor Rich Lyons.

The Miller Institute is proud to announce the awards for Professorship terms during the 2025-2026 Academic Year. These outstanding scientists pursue their research, following promising leads as they develop. The Visiting Miller Professors join faculty hosts on the Berkeley campus for collaborative research interactions.

Miller Professorship Awards

James Analytis
Physics

Roland Bürgmann
EPS

Eugene Chiang
Astro/EPS

Ronald Cohen
Chemistry

Shirshendu Ganguly
Statistics

Hernan Garcia
MCB/Physics

Jay T. Groves
Chemistry

Lin He
MCB

Na Ji
NeuroSci/Physics

Daniel Nomura
Chemistry/MCB

Sung-Jin Oh
Mathematics

Albert Ruhi
ESPM

Kimberley Seed
PMB

Michi Taga
PMB

Wenjun Zhang
CBE

Miller Fall Reception

Kathryn A. Day Miller Postdoctoral Fellow Molly McFadden with her host John Hartwig

Miller members at the Fall reception held at the Tilden room on campus

Miller Fellows Thuy-Duong (June) Vuong, Ian Waudby-Smith and former Miller Fellow Michael Celentano

> CONTINUED FROM PAGE 4 [Miller Research Competitions: Awards]

Visiting Miller Professorship Awards

Christl Donnelly
IB
Host: Michael Boots
Home Institution: University of Oxford

Alain Goriely
ME
Host: Oliver O'Reilly
Home Institution: University of Oxford

Jordan Horowitz
Physics
Host: Hernan Garcia
Home Institution: University of Michigan

Thomas Juenger
PMB
Host: Benjamin Blackman
Home Institution: The University of Texas at Austin

Anat Levin
EECS
Host: Laura Waller
Home Institution: Technion

Wencai Liu
Mathematics
Host: Svetlana Jitomirskaya
Home Institution: Texas A&M

Suliana Manley
Bioengineering
Host: Daniel Fletcher
Home Institution: Swiss Federal Institutes of Technology (EPFL)

Liviu Mirica
Somorjai Visiting Miller Professor
Chemistry
Host: Don Tilley
Home Institution: University of Illinois Urbana-Champaign

Caroline Muller
EPS
Host: David Romps
Home Institution: Institute of Science and Technology Austria (ISTA)

Cheng-Wei Qui
MSE
Host: Junqiao Wu
Home Institution: National University of Singapore

Jared Rutter
MCB
Host: Andrew Dillin
Home Institution: University of Utah

Paul Selvin
Neuroscience
Host: Ehud Isacoff
Home Institution: University of Illinois Urbana-Champaign

2024 Annual Fall Dinner

Executive Committee member Nicole King and guest speaker, Advisory Board member Professor Scott Edwards

Miller Senior Fellow Ken Ribet, Miller Fellow Matt Kustra, guest Elizabeth Feldstein, Miller Fellows Mark Carrington and Ian Waudby-Smith

Miller Senior Fellow Hitoshi Murayama, Executive Director Chung-Pei Ma, Chancellor Rich Lyons and Miller Professor Raphael Busso

Gifts to the Miller Institute

The Miller Institute gratefully acknowledges the following contributors to our programs in 2024. With your generosity, the Miller Institute is able to continue to support basic research in science at UC Berkeley.

Kathryn A. Day Miller Postdoctoral Fellowship Fund

The Kathryn A. Day Miller Postdoctoral Fellowship was established with a generous gift by Nobel Laureate Professor Randy Schekman and Professor Sabeeha Merchant to honor Kathy Day, who served as the Chief Administrative Officer at the Miller Institute for Basic Research in Science from 1989 - 2019. The purpose of the Fund is to provide an annual stipend, benefits and a research fund to a postdoctoral researcher at the Miller Institute who has demonstrated efforts towards community building and outreach in support of science.

Rachel Akeson
Anonymous
Justin M. Brown
Qian Chen
John Clarke
Kathryn A. Day (2)
Phyllis Day
Dmitry Dolgopyat
Alex Filippenko
Eric & Christina Ford
Yu He
Michael Hochberg
John E. Hunter III
Rita Huybrechts & Glen Price
Thomas Juenger
Yasuyuki Kawahigashi (2)
Michael Manga & Susan Storch
Frederick A. Matsen
Suzanne & Christopher McKee
Thomas Pollard
Ken Ribet & Lisa Goldberg (2)
Jasper Rine and Cynthia Kenyon
Stephen Suh
Jesse Thaler (12)
Rebecca Bishop White

Miller Institute for Basic Research in Science General Fund

The Miller Institute for Basic Research in Science is dedicated to the encouragement of creative thought and the conduct of research and investigation in the field of pure science. Contributions to this fund will support the four programs of the Miller Institute: the Miller Research Fellowship, the Miller Professorship, the Visiting Miller Professorship, and the Miller Senior Fellowship.

Anonymous
Steven & Diana Brenner
Justin M. Brown
William Craig Carter
Alex Filippenko
Iswar Hariharan
Gilbert & Judith Hawkins
Ronald Johnson
Russell Jones
Judith Klinman & Mordecai Mitnick
Andrew Ogg
Deborah Steinberg O.D. (IHO Megan Perin)
David Weinberg
Norman Yao

Miller Fellowship Program Development Fund

The Miller Fellowship Program Development Fund provides an annual stipend, benefits, and research support to young researchers at Berkeley. The program gives researchers the chance to explore ideas in a stimulating and supportive environment.

John E. Hunter III

Recent Publications

Omar Yaghi (Visiting Miller Professor 2009) co-authored a paper **"Carbon dioxide capture from open air using covalent organic frameworks,"** published in *Nature*. The discovery describes a new porous material, known as a covalent organic framework (COF), that efficiently removes carbon dioxide from the air and can be continuously reused.

Bin Yu's (Miller Professor 2004, 2016-2017) book with Rebecca Barter **"Veridical Data Science: the practice of responsible data analysis and decision-making"** was published in print and is available free online (https://mitpress.mit.edu/97802620.../veridical-data-science/ and a free on-line version is at https://vdsbook.com/.)

Inbal Ben Ami Bartal's (Miller Fellow 2014-2017) article on the complex affective and cognitive abilities of rats was published in the *Science special edition*.

Nicole King's (Miller Professor 2018-2019, Executive Committee 2023-2026) research on Mono Lake's unique choanoflagellates as models for early life forms that existed in ancient oceans that's offering glimpses into the evolutionary history that eventually gave rise to animals, was published by *The Brighter Side of News.*

An article entitled "Next-generation CRISPR-based gene-editing therapies tested in clinical trials" in *Nature Medicine* discusses the advances in Sickle-cell disease treatment, the world's first approved CRISPR–Cas9 therapy. *"I'm hopeful that gene-editing therapies can start to become a new standard of care, transforming the landscape of modern medicine and improving countless lives,"* said **Jennifer Doudna** (Miller Senior Fellow 2017).

This fall, the Miller Institute welcomed the **3rd Kathryn A. Day Miller Postdoctoral Fellow, Mark Carrington (2024-2027).** This special award is intended for an early-career scientist who, in addition to excelling in their research, has also engaged in community building and outreach in support of science.

Next Steps

Kelian Dascher-Cousineau
Assistant Professor
Geosciences
Utah State University

Aaron Joiner
Postdoc @ Hurley lab
Molecular & Cell Biology
UC Berkeley

Elena Zavala
Assistant Professor
Forensic Medicine
University of Copenhagen

Xueyue (Sherry) Zhang
Assistant Professor
Applied Physics and Applied Mathematics
Columbia University

Fall 2024 Miller Events

Members of our new Fellowship cohort (2024-2027) joined our Executive Committee, Institute staff and several Hosts in September for a **New Fellows Retreat** at the Women's Faculty Club. **Top Row** (L to R): Ian Waudby-Smith, Matt Kustra, Mohammad Farhat, Jonathan Wolf, Calvin Leung, Kate Reidy & Mark Carrington. **Bottom Row** (L to R): Thuy-Duong (June) Vuong, Katie Fraser, & Sarah Zeichner

Non-Profit
Organization
U.S. Postage
PAID
University of
California,
Berkeley

University of California, Berkeley
Miller Institute for Basic Research in Science
206B Stanley Hall
Berkeley, CA 94720-5190
miller.berkeley.edu

1-36299-24810-44

Miller Institute News : Winter 2025
Please send address corrections to:
miller_adm@berkeley.edu

Make a Gift

Private donations are becoming an increasingly significant resource for the Miller Institute. Your personal investment in support of the future of the Miller Institute will be greatly appreciated.

Join Miller friends and alumni in contributing to this important endeavor by logging on to **miller. berkeley.edu/gift** to help support the indepen-dent research of the Miller Institute members.

SUBSCRIBE
TO OUR NEWSLETTER

Subscribe to our digital newsletter by visiting **miller.berkeley.edu/news/newsletters** and clicking on the **SUBSCRIBE** button!

For More Information:
Staff: Emily Birman, Clara Duman, Donata Hubert, Hilary Jacobsen & Vrinda Khanna

Fall 2024 Miller Events

Miller members enjoyed the **Berkeley Marina Guided Shoreline Walk** led by **Robert Dudley** (Miller Professor 2024-2025)